Structural reliability of a RC beam considering different preliminary designs

Influência do lançamento estrutural na confiabilidade de vigas de concreto armado

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ABSTRACT

In the last decades, most of the structural designs around the world have been made by employing semi-probabilistic approaches based on partial safety factors. These practical methods allow the engineer to deal with the safety of structures without the need to directly address the uncertainties involved. The simplicity of these approaches depends on the fact that the same values of the safety factors must be applicable to a large number of different cases; this typically leads to dispersion in the structural reliability levels. The present paper aims at quantifying such dispersion within a specific scenario, regarding RC structures, in a way different from those found in the literature. It focuses on a simple residential building, for which seven preliminary design solutions are proposed by different engineers, and seven different structures are obtained from these solutions, considering the specifications given by the Brazilian structural design code ABNT NBR 6118:2014. The structural reliability indexes of a continuous beam, common to all solutions, are assessed and compared. The results indicate that, even in this specific case, the variability related to the safety levels is significant. Furthermore, in some cases the reliability index is found to be below the target levels provided by international structural standards.

Keywords: structural reliability, preliminary structural design, reinforced concrete beams, Monte Carlo simulation.
RESUMO
Atualmente, a maioria dos projetos estruturais ao redor do mundo tem sido elaborados a partir de abordagens semi-probabilísticas baseadas em fatores de segurança parciais. Estes métodos permitem aos engenheiros tratarem a segurança das estruturais de maneira mais prática, sem a necessidade de abordar diretamente as incertezas envolvidas. A simplicidade dessas abordagens está relacionada ao fato dos mesmos fatores de segurança serem utilizados para um amplo número de casos diferentes estruturais, desde elementos até cargas com intensidades e distribuições distintas. O mencionado leva a dispersão dos níveis de confiabilidade estrutural. O presente artigo visa quantificar tal dispersão dentro de um cenário relacionado a estruturas de concreto armado, analisando o problema de uma forma distinta dentre às usualmente encontradas na literatura. Considera-se um projeto arquitetônico de uma casa térrea, no qual são obtidos sete diferentes lançamentos estruturais por diferentes engenheiros, considerando as especificações da norma brasileira de projeto estrutural ABNT NBR 6118:2014. Os índices de confiabilidade estrutural de uma viga contínua, comum a todas as soluções, foram avaliados e comparados. Os resultados indicam que, mesmo para esse caso específico, a variabilidade relacionada aos níveis de segurança da estrutura é bastante significativa. Além disso, em alguns casos, o índice de confiabilidade atingido é inferior aos índices mínimos recomendados pelas normas estruturais internacionais.

Palavras-chave: confiabilidade estrutural, lançamento estrutural, vigas de concreto armado, simulação de Monte Carlo.

1 INTRODUCTION
The basic objective of structural design is to develop structural systems which satisfy a number of constraints. These constraints can be categorized into: those which must be fully satisfied, the so-called hard ones, such as functionality; and those which may be partially satisfied, the soft ones, such as cost (Harty and Danaher [2]). Current practice of reinforced concrete (RC) structural design usually defines hard constraints in terms of requirements or recommendations, given by structural design codes such as the Brazilian Standard ABNT NBR 6118 [1] and the American standard ACI 318 [3]. The safety of the resulting structures is directly dependent on these constraints.

In the last decades, most of the structural designs around the world have been made by employing semi-probabilistic approaches based on partial safety factors, presented in such design codes. These approaches allow the engineer to deal with the safety of the structures without the need to directly address the uncertainties involved; they are still considered more practical than other methods which employ probabilistic assessments to take into account the uncertainties. The simplicity and applicability of the semi-probabilistic approaches is highly dependent on the fact that it must be possible to apply the same values of the safety factors to a large number of different cases and always
obtain acceptable safety levels. Safety factors are usually calibrated, in order to reduce the dispersion of the reliabilities related to different structures and to keep the reliability indexes within acceptable levels (Nowak [4]; Ellingwood [5]; Sorensen et al. [6]; Szerszen and Nowak [7]; Beck and Souza Jr. [8]; Kotes et al. [9]). Although the variability is reduced by the calibration process, it still exists and is usually not neglectable.

The literature presents many papers illustrating how different reliability levels are achieved, even when different structures are designed according to the same requirements and using the same partial safety factors (Szerszen et al. [10]; Stucchi and Santos [11]; Beck and Doria [12]; Beck et al. [13]; Chaves et al. [14]; Santos et al. [15]; Nogueira and Pinto [16]). On the other hand, only a few papers in the literature deal with the fact that the preliminary design of a given structure also have a significant impact on its final reliability. According to Harty and Danaher [2], the designer’s task in preliminary structural design involves activities and decisions that are heuristic in nature and rely more on experience and judgment than on computation.

An investigation about the effect of uncertainties related to knowledge and experience of the structural engineer, under a reliability point of view, may be found for example in Fröderberg and Thelandersson [17]. In this paper, the authors considered such uncertainties by introducing a term called Engineering Modeling Uncertainty, and showed that they have a large effect on structural safety.

In the present paper, the effect of the preliminary design on the structural safety is investigated in a quite different way. Seven preliminary design solutions are proposed by different engineers for a given residential building. After that, different structures are obtained from these solutions considering the design specifications presented by the Brazilian structural design code ABNT NBR 6118:2014 [1]. Finally, the structural reliability indexes of a continuous beam, common to all solutions, are assessed and compared.

The remainder of this paper is organized as follows. In Section 2, some basics concepts of structural reliability are presented. A brief description of the application of structural reliability in the context of reinforced concrete structures, as performed herein, is presented in Section 3. The chosen residential building as well as its structural design solutions are described and illustrated in Section 4 and results associated with each solution, in terms of reliability indexes, are presented in Section 5. Finally, some concluding remarks are drawn in Section 6.
2 STRUCTURAL RELIABILITY THEORY

The existence of uncertainties which affect structural systems imply the possibility of undesirable structural responses, usually called failures. It has become standard to differentiate between uncertainties due to inherent natural variability, often denoted aleatory uncertainty, model uncertainties and statistical uncertainties, referred to as epistemic uncertainties (Faber [18]). The main goal of structural reliability is typically to determine failure probabilities, taking into account these uncertainties. Mathematical representations of uncertain parameters are commonly obtained by means of random variables, and the random variables related to a given reliability problem may be grouped into a vector $X$. The boundary between desirable and undesirable structural responses may be given by a limit state function, $g(X)$, leading to the definition of a failure domain, $\Omega_f$, and a safe domain, $\Omega_s$:

$$
\Omega_f = \{ x | g(x) \leq 0 \}
$$

$$
\Omega_s = \{ x | g(x) > 0 \}.
$$

Each limit state relates to a particular type of structural failure (Melchers and Beck [19]). In general, the probability of failure of a structural system ($P_f$) can be computed by the following integral:

$$
P_f = \int_{\Omega_f} f_X(x) dx
$$

Where:

$f_X(x)$ is the joint probability density function of the vector of random variables.

Equation (2) can be solved, for example, by transformation methods such as the First and Second Order Reliability Methods (FORM and SORM, respectively) (Madsen et al. [20]; Haldar and Mahadevan [21]) or by simulation methods, such as the simple Monte Carlo simulation (SMC) (Haldar and Mahadevan [21]). Solution by simple Monte Carlo simulation employs a so-called indicator function, $I[x]$, which equals one if failure occurs, i.e. if $g(x) \leq 0$, or zero otherwise. Introducing the indicator function into the equation, integration may be performed over the entire domain, $\Omega$, and in an approximate
manner if a finite number of samples of $X$ is generated according to the joint distribution $f_X(x)$:

$$P_f = \int_{\Omega} \mathbb{I}[g(x) \leq 0] f_X(x) \, dx \approx \frac{1}{N} \sum_{j=1}^{N} \mathbb{I}[g(x_j) \leq 0]$$

(3)

Where:

$N$ is the total number of samples.

In general, the greater the number of samples adopted, the more accurate the estimate of the failure probability. On the other hand, the smaller the failure probability to be estimated, the higher the number of samples required to achieve a given accuracy. As the failure probabilities in structures are very small, in the order of $10^{-3}$ to $10^{-6}$, the total number of samples to achieve a value with an acceptable confidence interval is quite high (Ditlevsen and Madsen [22]).

In practice, it is very common to represent the safety of a structure by means of the associated reliability index, $\beta$, given by Equation (4), as it is done in the present paper.

$$\beta = -\Phi^{-1}(P_f)$$

(4)

Where:

$\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function in the standard normal space.

### 3 REINFORCED CONCRETE STRUCTURES AND RELIABILITY ANALYSIS

Many applications of structural reliability analysis to reinforced concrete structures may be found in the literature, especially in the last decades (Awruich and Gomes [23]; Novaes et al. [24]; Zhang [25], Porto et al. [40], Furst [41]). A lot of studies have been devoted to the particular case of reliability analysis of structures designed according to design codes.

For example, Mohamed et al. [26] studied RC columns considering different concrete strengths, slenderness indexes and longitudinal reinforcements, and stated that there is a lack of uniformity in safety of these elements when they are designed by the
Eurocode 2 “Design of concrete structures”. Eamon and Jensen [27] conducted a reliability analysis for various reinforced concrete columns exposed to fire, designed according to ACI 318-08. The study presents a significant variation in reliability for RC members of the same importance level, designed using the same code criteria. Baji et al. [28], compared reliability indexes related to ductility requirements for RC beams in concrete design codes. They concluded that the probabilistic nature of ductility is not adequately addressed by the current state of the art; in such a way that there is a larger disparity among codes for the reliability of ductility than there is for strength.

Some studies about the Brazilian Standard ABNT NBR 6118 [1] have also been presented in the literature. Santos et al. [15] for example, analyzed the reliability indexes of reinforced concrete beams, considering different longitudinal reinforcements and live-to-dead load ratios. They found out that, in some cases, the safety levels obtained may be lower than the limits given by the American standard ACI 318 [3] and by the Eurocode EN 1990 [29] even for usual load ratios. They also found that the Brazilian code does not provide a uniform level of reliability, as also presented by other references (Stucchi and Santos [11]; Nogueira and Pinto [16]; Stucchi and Santos [30]).

As previously stated, reliability analyses are based on proper descriptions of uncertainties as well as on sufficiently accurate descriptions of failure; the latter is typically done by using the concept of limit states. Limit states have the objective of ensure that the structure presents a satisfactory performance, both in situations of greater risk and in normal service conditions. The Brazilian code for design of reinforced concrete structures, ABNT NBR 6118 [1], classifies the limit states into two types: ultimate limit states, corresponding to the structural safety requirements; and serviceability limit states, associated to the good structural performance under normal load conditions. The structure is usually designed for ultimate limit states, where the ultimate load combinations take place, and checked for the serviceability limit states (Kong and Evans [31]).

Equation (5) describes the ultimate normal combination, which leads to the load values to be used in design, Fd. To compute these values, actions which may have significant effects on the structure are first classified into permanent, variable and accidental actions, according to their duration, variation in time and probability of occurrence during the lifespan of the structure, as can be seen in ABNT NBR 6118 [1].
\[ F_d = \gamma_g F_{gk} + \gamma_q \left( F_{q1k} + \sum \Psi_0 F_{qjk} \right) \]  

Where:

- \( F_{gk} \) is the characteristic value of the permanent actions;
- \( F_{q1k} \) is the characteristic value of the main variable action;
- \( \Psi_0 F_{qjk} \) is the characteristic value of the secondary variable actions, \( F_{qjk} \), reduced by the factor \( \Psi_0 \);
- \( \gamma_g \) and \( \gamma_q \) are the partial safety factors related to permanent and variable actions, respectively. For reinforced concrete structures, both partial safety factors are usually given as \( \gamma_g = \gamma_q = 1.4 \) in the design code (ABNT NBR 6118 [1]).

The present paper focuses on beams of rectangular cross-sections, and both the designs and reliability analyses are made under the assumption that the beams are subjected to pure bending. The beams are assumed singly reinforced, so that there is no contribution related to compression reinforcements. In this case, the strength of the reinforced concrete beam, in terms of flexural moments, may be derived from the equilibrium equations taking a cross section of the structural element as illustrated in Figure 1.

Figure 1 Equilibrium of the RC beam of rectangular cross section on strain domain 3, valid to normal strength concretes (Group I)

This figure also presents the equivalent rectangular stress block, a simplified representation of the stress distribution in the compressed concrete zone, recommend by the ABNT NBR 6118 [1]. The resultant force of compression in the concrete and the resultant force in the longitudinal tensile reinforcement, \( R_{cc} \) and \( R_{st} \), respectively, are described by Equations (6) and (7).
\[ R_{cc} = \alpha_c \frac{f_{ck}}{\gamma_c} (0.8 \times b) \quad (6) \]

\[ R_{st} = A_s \frac{f_{yk}}{\gamma_s} \quad (7) \]

Where:

\( A_s \) is the cross-sectional area of the longitudinal tensile steel reinforcement; \( f_{yk} \) and \( f_{ck} \) are the characteristic yield strength of steel and the characteristic compressive strength of concrete, respectively; \( \gamma_s \) and \( \gamma_c \) are the partial factors for reinforcement and concrete strength, respectively; \( \alpha_c \) is a parameter of reduction of the compressive strength of concrete, which equals 0.85 for \( f_{ck} \leq 50 \text{ MPa} \); \( x \) is the neutral axis position and \( b \) is the cross-section width. Equation (6) is applied to evaluate the compressive force, in the case of normal-strength concretes. The internal bending moment is obtained by multiplying \( R_{st} \) by the distance between the resultant internal forces, \( z \).

The required area of tension reinforcement is given by Equation (8) as a result of the equilibrium of forces in the cross section.

\[ A_s = \frac{M_{sd}}{\frac{f_{yk}}{\gamma_s} (d - 0.4x)} \quad (8) \]

Where:

\( d \) is the effective depth of longitudinal reinforcement and \( M_{sd} \) is the design bending moment.

Finally, from the equilibrium of axial forces and moments in the cross section, the design flexural strength \( (M_{Rd}) \) of a beam of rectangular cross section is given by Equation (9).

\[ M_{Rd} = A_s \frac{f_{yk}}{\gamma_s} \left( h - d' - 0.5 \frac{A_s f_{yk}}{\gamma_s 0.85b f_{ck} \gamma_c} \right) \quad (9) \]

Where:

\( h \) is the height of the cross-section and \( d' \) is the distance from the bottom face of the concrete to the center of gravity (CG) of the reinforcement.
This equation is valid only for the so-called strain domains 2 and 3, where the yielding of the steel is supposed to take place in the ultimate limit state. Furthermore, as recommend by the ABNT NBR 6118 [1], a constraint is imposed on the relative position of the neutral axis, given by \( x/d \leq 0.45 \), as an attempt to ensure sufficient ductility.

For the reliability analysis, the resistance in terms of flexural strength, which now becomes a random variable, \( R(X) \), is obtained by employing unitary partial safety factors and considering one or more of the remaining parameters as random variables. On the other hand, the load variable, which is also a random variable, \( S(X) \), is a function of the permanent, variable and accidental loads; in this case, a permanent moment, \( M_G \), a variable moment, \( M_Q \), and an accidental bending moment related to the wind, \( M_W \). The resulting limit state function, \( g(X) \), where \( X = \{ A_s, f_y, f_c, b, h, d', M_G, M_Q, M_W, \theta_R, \theta_S \} \) is the vector of random variables, is given by Equation (10) (Santos et al. [15]).

\[
g(X) = R(X) - S(X) = 0_R A_s f_y \left( h - d' - 0.5 \frac{A_s f_y}{0.85 f_c b} \right) - 0_S (M_G + M_Q + M_W) \tag{10}
\]

Where:

\( \theta_R \) and \( \theta_S \) are the model uncertainties for the bending moment capacity and for the load, respectively, obtained from the literature [15]. The statistical parameters and distributions of each random variable, as adopted herein, are summarized in Table 1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Variable</th>
<th>Simb.</th>
<th>Dim.</th>
<th>Distribution</th>
<th>( \mu_X )</th>
<th>C.O.V.</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actions</td>
<td>Permanent M_G</td>
<td>kN.m</td>
<td>Normal</td>
<td>1.05 M_G</td>
<td>0.10</td>
<td></td>
<td>[15]</td>
</tr>
<tr>
<td></td>
<td>Variable M_Q</td>
<td>kN.m</td>
<td>Gumbel</td>
<td>0.934 M_Q</td>
<td>0.20</td>
<td></td>
<td>[15]</td>
</tr>
<tr>
<td></td>
<td>Wind M_W</td>
<td>kN.m</td>
<td>Gumbel</td>
<td>0.90 M_W</td>
<td>0.34</td>
<td></td>
<td>[8]</td>
</tr>
<tr>
<td>Strength of</td>
<td>Concrete f_c</td>
<td>MPa</td>
<td>Normal</td>
<td>1.196 f_c</td>
<td>0.15</td>
<td></td>
<td>[15]</td>
</tr>
<tr>
<td>materials</td>
<td>Reinforcement f_y</td>
<td>MPa</td>
<td>Normal</td>
<td>1.089 f_y</td>
<td>0.05</td>
<td></td>
<td>[15]</td>
</tr>
<tr>
<td>Geometry</td>
<td>Cross-sectional area A_s</td>
<td>cm²</td>
<td>Normal</td>
<td>A_s</td>
<td>0.015</td>
<td></td>
<td>[15]</td>
</tr>
<tr>
<td></td>
<td>of longitudinal tension</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Width of the section b</td>
<td>cm</td>
<td>Normal</td>
<td>b</td>
<td>0.067</td>
<td></td>
<td>[11], [15]</td>
</tr>
<tr>
<td></td>
<td>Height of the section h</td>
<td>cm</td>
<td>Normal</td>
<td>h</td>
<td>0.045</td>
<td></td>
<td>[11]</td>
</tr>
<tr>
<td></td>
<td>Distance from</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>extreme tension fiber</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>of concrete to CG of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>the tension</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>reinforcement d'</td>
<td>cm</td>
<td>Lognormal</td>
<td>d'</td>
<td>0.27</td>
<td></td>
<td>[11]</td>
</tr>
<tr>
<td>Models</td>
<td>Load uncertainties ( \theta_S )</td>
<td>-</td>
<td>Lognormal</td>
<td>1.0</td>
<td>0.05</td>
<td></td>
<td>[11]</td>
</tr>
</tbody>
</table>
In a continuous beam subject to pure bending, failure may be defined in a number of ways. In the present paper, three different cases are considered:

- Failure occurs when the positive bending moment in the critical midspan reaches the bending strength;
- Failure occurs when the negative bending moment in the region of the critical support reaches the bending strength;
- Failure occurs when the critical positive and/or the critical negative bending moments reach the respective bending strengths. In this case, the beam is seen as a series system and the respective failure probability is be given by:

$$P_{f\text{ system}} = P[A \cup B] = P[A] + P[B] - P[A \cap B]$$ \hspace{1cm} (11)

Where:

$P[A]$ and $P[B]$ are failure probabilities related the critical positive and negative bending moments and $P[A \cap B]$ is related to the intersection between the first two failure modes. The critical positive and negative bending moments are understood as those with maximum absolute value along the beam.

One notes that the combination of possible failure modes is a closer representation of the real failure of the beam. Furthermore, by considering a plastic analysis, the collapse of the beam takes place only when a sufficient number of plastic hinges develop. However, target values of reliability indexes available in the literature are usually related to the analysis of the most critical cross-section only, which justifies the investigation of the reliability indexes related to the critical cross-sections alone. In a simplified manner, this paper also presents the combination of the two failure modes as an indicative of the system failure probability.

In all cases studied herein, the software Risk Tools, developed by Mahsuli and Haukaas [32], is used to evaluate failure probabilities and reliability indexes. Simple Monte Carlo simulation is employed, by defining a target coefficient of variation of 5% for the failure probability estimate and increasing the number of samples until this coefficient of variation is achieved, as indicated in Table 2.
Table 2 Monte Carlo sample size of the structural design solutions for the different failure modes

<table>
<thead>
<tr>
<th>Design solution</th>
<th>Critical positive</th>
<th>Critical negative</th>
<th>Series system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.22 \times 10^6$</td>
<td>$9.64 \times 10^5$</td>
<td>$6.88 \times 10^3$</td>
</tr>
<tr>
<td>2</td>
<td>$9.00 \times 10^7$</td>
<td>$6.00 \times 10^7$</td>
<td>$3.00 \times 10^7$</td>
</tr>
<tr>
<td>3</td>
<td>$1.08 \times 10^7$</td>
<td>$4.20 \times 10^6$</td>
<td>$2.97 \times 10^6$</td>
</tr>
<tr>
<td>4</td>
<td>$1.50 \times 10^8$</td>
<td>$1.94 \times 10^6$</td>
<td>$1.88 \times 10^6$</td>
</tr>
<tr>
<td>5</td>
<td>$7.00 \times 10^8$</td>
<td>$2.68 \times 10^6$</td>
<td>$2.84 \times 10^6$</td>
</tr>
<tr>
<td>6/7</td>
<td>$6.43 \times 10^6$</td>
<td>$2.37 \times 10^7$</td>
<td>$5.60 \times 10^6$</td>
</tr>
</tbody>
</table>

Source: Authors.

4 CASE STUDY AND STRUCTURAL DESIGN SOLUTIONS

The reference building considered herein is a single floor residential building (Figure 2), located in the city of Florianópolis, Brazil, whose floor plan is illustrated in Figure 3a. The environmental aggressiveness is considered to be moderate, following the classification provided by ABNT NBR 6118 [1], which relates to an urban area, with low risk of structural deterioration. The structural elements are identified following the location plan provided in Figure 3b, and are designed by considering a concrete class C25 and a reinforcement steel class CA-50, with $f_{ck} = 25$ MPa and $f_{yk} = 500$ MPa, respectively. As is common in this kind of building, some dimensions of the structural elements are adopted as equal to the width of the wall, due to aesthetic purposes. In this paper, the interior wall thicknesses are all taken as equal to 15 cm.
The structure is subject to live, dead and wind loads. The live load is related to access for maintenance and cleaning of the roof, and is taken as equal to 0.5 kN/m², following ABNT NBR 6120 [33], while the dead load refers to the self-weight of the roof,
slabs and beams, and is calculated for each case. The self-weight of the roof and the superimposed dead load are assumed equal to 0.9 kN/m² and 0.25 kN/m², respectively. The wind load is estimated by following the procedures specified by ABNT NBR 6123 [34], assuming a basic wind velocity of 43 m/s, which leads to a wind pressure of about 0.708 kN/m².

After defining the basic parameters of the building, some structural engineers from Florianópolis were asked to perform preliminary designs for it, giving rise to a total of seven different structural design solutions, as shown in Figure 4. Their task was basically to define locations, orientation and some dimensions of beams and columns, in an attempt to create structural systems with small costs and satisfactory levels of safety.

Figure 4 Structural design solutions for the architectural design of reference obtained by different structural engineers

Source: Authors.
For the sake of simplicity, the present paper focuses on the reliability analysis of a single continuous beam, which spans over the I axis, between axes 2 and 8 (see Figure Error! Fonte de referência não encontrada.b), and is common to all design solutions.

Internal forces on structural elements, as well as remaining dimensions of the cross-sections and minimum reinforcement ratios required, are determined by employing a structural model developed using the commercial structural design software Eberick (AltoQi [35]), which essentially follows the Brazilian design codes. For the longitudinal reinforcement, four different possible diameters are considered: 6.3, 8.0, 10.0 and 12.5 mm. Decision about the diameters is made by taking into account the reinforcement ratio required, as well as construction concerns. For example, it is common practice to avoid diameters which would lead to too many bars. Table 3 presents the values of bending moments, steel reinforcement areas and reinforcement ratio, distances from the bottom face of the concrete to the CG of the reinforcement (or upper face in the case of negative bending moments) and cross-sectional dimensions for all design solutions; the sixth and seventh solutions are grouped since they resulted essentially the same. For all cases, results for two critical points along the beam are provided; they are related to the positive and negative bending moments with maximum absolute values. Consequently, it is assumed that failure may occur in one of the mid-spans of the beam and/or over one of its supports. Therefore, $A_{s1}$ refers to the required area of bottom longitudinal reinforcement in the midspan subject to the largest bending moment and $A_{s2}$ refers to the required area of upper longitudinal reinforcement in the region around the support where the negative bending moment assumes its maximum absolute value, as indicated in Figure 5.

Table 3 Average values of the critical section loads and the parameters of RC beam I designed from different structural design solutions considering C25 concrete and CA-50 steel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Structural Design Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$M_{el}$ (kN.m)</td>
<td>5.18</td>
</tr>
<tr>
<td>$M_{el}$ (kN.m)</td>
<td>0.61</td>
</tr>
<tr>
<td>$M_{el}$ (kN.m)</td>
<td>0.51</td>
</tr>
<tr>
<td>$A_{s1}$ (cm²)</td>
<td>1.005</td>
</tr>
<tr>
<td>$\rho_{s1}$</td>
<td>0.27%</td>
</tr>
<tr>
<td>$d_{l1}$ (cm)</td>
<td>4.00</td>
</tr>
<tr>
<td>$M_{el2}$ (kN.m)</td>
<td>10.57</td>
</tr>
<tr>
<td>$M_{el2}$ (kN.m)</td>
<td>1.21</td>
</tr>
</tbody>
</table>
5 RELIABILITY ANALYSIS RESULTS

In this section, reliability indexes for the beam under study are obtained and compared considering all structural design solutions previously proposed; they are also compared to target values recommended by some international design codes (ACI 318 [3]; EN 1990 [29]). Material costs related to the beam and the adjacent columns are presented as well.

The reliability indexes for the RC beam are shown in Figures 6, 7 and 8, for the three different cases in terms of failure modes. Due to computational limitations concerning a maximum number of simulations, the very small failure probability related to the RC beam of the fifth design, under maximum positive moment, could not be directly evaluated. This does not affect the discussions and conclusions presented herein, since the reliability index for this case was estimated to be higher than 5.0.
Figure 6 Reliability indexes achieved for each structural design solution, considering failure by the critical positive bending moment.

Figure 7 Reliability indexes achieved for each structural design solution, considering failure by the critical negative bending moment.

Source: Authors.
When dealing with failure by maximum positive moment, in the critical midspan, Figure 6, the ACI index target of 3.5, as indicated by Szerszen and Nowak [7], is met by all design solutions; however, the first design solution does not achieve the reliability index target of 3.8, recommended by the Eurocode [29]. A considerable variation of reliability is found for this failure mode, with a minimum reliability index of 3.57 for the first structural design solution and a maximum greater than 5.00 for the fifth.

Results related to failure by the maximum absolute negative moment are presented in Figure 7. In this case, the first design solution led to a safety level which is below both target values. All the other solutions achieved reliability levels above the value indicated by the ACI 318 [3] but three of them, the third, fourth and fifth, were below 3.8.

Combination of the two previous failure modes as a series system leads to the results shown in Figure 8, which are similar to those related to the negative moment but with less reliability, in general. The reliability level for the first design solution is below both targets, while the reliability indexes of the third, fourth and fifth design solutions are below the target value given by the Eurocode [29].

Note that many variables contribute to these results, so that these results can hardly be anticipated without performing the reliability analysis. For example, by taking a look at Table 3 it is seen that the first design solution renders the larger steel reinforcement ratio for the negative moment, leading to less variability of the overall material strength; but this is also the case with the smallest reliability level, according to Figure 7.
ratios, as well as other parameters, change according to the position of the structural members, their dimensions and reinforcement ratios, their interactions with other structural members, and so on. These different scenarios induce variabilities in the safety levels obtained, as can be seen in Figures 6, 7 and 8, even if the same procedures and safety factors are employed in the design.

Another concern, related to the “price of safety”, could be raised in this problem. Is the beam from the first design solution, the less safe, also the cheapest one? In an attempt to answer this question, the total cost of the beams together with their adjacent columns is estimated, by adopting the minimum reinforcement ratio of 0.15% for the columns, as specified by ABNT NBR 6118 [1].

Table 4 presents the volumes of concrete for the beam and the columns, Vbeam and Vcolumns, respectively, the total weight of steel, Wsteel, and the costs for each design solution. Unitary costs were obtained from the unit costs table of the National System of Costs Survey and Indexes of Construction (SINAPI), in September 2017 for the Santa Catarina state, Brazil, which was provided by CAIXA [36]. The average cost for CA-50 steel bars was R$3.85 per kilo and for the C25 concrete the cost (without pumping) was R$307.67 per cubic meter.

Table 4 Costs and materials consumption for the frame containing the beam designed by different structural design solutions

<table>
<thead>
<tr>
<th>Design solution</th>
<th>Vbeam (m³)</th>
<th>Vcolumns (m³)</th>
<th>Wsteel (kg)</th>
<th>Cost (R$)</th>
<th>Relative cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.487</td>
<td>0.304</td>
<td>35.68</td>
<td>380.63</td>
<td>Reference</td>
</tr>
<tr>
<td>3</td>
<td>0.557</td>
<td>0.203</td>
<td>34.76</td>
<td>367.33</td>
<td>96.5%</td>
</tr>
<tr>
<td>6/7</td>
<td>0.417</td>
<td>0.304</td>
<td>37.45</td>
<td>366.05</td>
<td>96.2%</td>
</tr>
<tr>
<td>5</td>
<td>0.417</td>
<td>0.405</td>
<td>27.88</td>
<td>360.37</td>
<td>94.7%</td>
</tr>
<tr>
<td>4</td>
<td>0.348</td>
<td>0.405</td>
<td>28.49</td>
<td>341.30</td>
<td>89.7%</td>
</tr>
<tr>
<td>1</td>
<td>0.348</td>
<td>0.304</td>
<td>33.92</td>
<td>331.05</td>
<td>87.0%</td>
</tr>
</tbody>
</table>

Source: Authors.

The beam from the first design solution was found to be the cheapest one, and Figure 8 shows that this is the less safe too. The beam from the second design solution is shown to be the safer but also the most expensive. On the other hand, although the beam from the third design solution is more expensive than the one from the fifth solution, they have the same safety level. This shows that there is a relation between costs and safety, as expected; but this also shows that it is possible to spend more in an inefficient way, without increasing the reliability. This kind of observation led the path to reliability-based
structural optimization formulations, which have been widely applied in the literature in the last decades (Aoues and Chateaueneuf [37]; Mitropoulou et al. [38]; Beck et al. [39]).

6 CONCLUSIONS

In this paper, the variability in reliability of reinforced concrete beams subject to pure bending and designed according to the Brazilian standard ABNT NBR 6118 [1] was investigated in a specific scenario. Reliability indexes of a continuous beam from a residential building, considering seven preliminary design solutions proposed by different engineers, were assessed and compared. The results indicated that the application of the design procedures and the corresponding partial safety factors does not lead to uniform reliability levels, even when dealing with a single continuous beam of a given building. In other words, the decisions made during preliminary designs based on the experience and judgment of the structural engineer may lead to different safety levels, even for a single-floor residential building. Furthermore, in some cases the reliability indexes were found to be below acceptable levels indicated by international structural design codes, as already pointed out in other studies, presented by other papers from the literature (Stucchi And Santos [11]; Beck et al. [13]; Santos et al. [15]; Baji et al. [28]).

Finally, a brief discussion about the costs of the beams showed that greater costs do not necessarily mean more safety. Beams with similar costs achieved significantly different reliability levels. This is in accordance with the fact that a large number of applications of reliability-based structural optimization formulations have been seen in the literature in the last decades.

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