The art of fitting financial time series for some stock market indexes of some of the main emerging countries and for Mexico with stable Lévy Distributions

A arte de ajustar séries temporais financeiras para alguns índices bolsistas de alguns dos principais países emergentes e para o México com distribuições de Distribuições de Levy

DOI:10.34117/bjdv9n6-007

Recebimento dos originais: 26/05/2023
Aceitação para publicação: 02/06/2023

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ABSTRACT
This paper presents a statistical analysis of the logarithmic fluctuations of the returns of some of the main stock market indices. Samples were analyzed considering daily data covering a period from 01/03/2000 to 09/30/2022, and adjusted to different distributions. Goodness-of-fit tests were performed to quantitatively assess the quality of the estimate. Particular attention was paid to the impact of sample size on the estimated decay of the tail of the distributions. In this study, a forceful rejection of normality was obtained. On the other hand, the null hypothesis that the logarithmic fluctuations conform to a stable alpha Lévy distribution cannot be rejected considering a significance level of 5%.

Keywords: stock markets, high frequency fluctuations distribution, tail behavior, autocorrelations, distribution's stable.

RESUMO
Este artigo apresenta uma análise estatística das flutuações logarítmicas dos retornos de alguns dos principais índices do mercado de ações. As amostras foram analisadas considerando dados diários abrangendo um período de 01/03/2000 a 30/09/2022, e ajustadas para diferentes distribuições. Testes de qualidade de ajuste foram realizados para avaliar quantitativamente a qualidade da estimativa. Particular atenção foi dada ao impacto do tamanho da amostra na estimativa de decaimento da cauda das distribuições. Neste estudo, obteve-se uma rejeição contundente da normalidade. Por outro lado, a hipótese nula de que as flutuações logarítmicas obedecem a uma distribuição alfa de Lévy estável não pode ser rejeitada considerando um nível de significância de 5%.

Palavras-chave: mercados de ações, distribuição de flutuações de alta frequência, comportamento da cauda, autocorrelações, distribuição estável.

1 INTRODUCTION
Nowadays it is widely recognized that for the correct management of assets and prices (and the related investment risks) an adequate modeling for the distribution of returns on financial assets is required. For example, it is possible to see how the market directly shows an increase in factors such as price in scenarios that present a long prevalence memory and how likely price fluctuations are. However, the crucial difficulty of financial markets is their complexity since having a large number of internal elements that interact in a non-linear way, it is very sensitive to the action of external forces. Furthermore, the real challenge here is that the number of components in the system and
the details of their interactions and the external factors that act on them are hardly known in reality.

Within the field of physics, there is a long tradition of handling similar systems, the statistical description of many-particle systems developed in parallel with the statistical analysis of market dynamics. For example, taking into account the Central Limit Theorem (Bachelier, 1900) he assumed that performance on a given time scale is the consequence of many independent 'microscopic' events, which then lead to a normal distribution. Therefore, he modeled the dynamics of it as an uncorrelated walk with identically distributed independent random variables (Gaussian), that is, as a Brownian motion. Since then, the Gaussian assumption for the distribution of returns has been used frequently in finance as it is one of the key assumptions behind the classical Black-Scholes formula for stock prices (Black-Scholes, 1973). which is based on a continuous-time Wiener process or on appropriate discrete-time versions such as binomial trees.

Although the results provided by the normal distribution in analytical calculation are very valuable, empirical studies show that the distribution of yield fluctuations has a heavier tail than that presented in a Gaussian model. To illustrate this fact, Graph 1 shows the histogram of the daily logarithm differences of the Mexican IPC index from January 30, 2000 to September 30, 2022. Clearly, it is observed that large events are very frequent in the data, an important fact that is largely underestimated by a Gaussian process that is itself of paramount importance for financial management. It is notable that this characteristic is present within various financial markets.

Heavy-tailed distributions are commonly described by a power law (at least considering a range of scales), which in turn implies scale invariance, a distinct form for fractals. Studies have shown that fractals are a geometric pattern within many natural systems. Finally, in many of these systems they may be in a self-organized critical state. This could also be the case for other dynamical systems outside the realm of science. In his pioneering analysis of cotton prices, Mandelbrot (Mandelbrot, 1963), founder of fractal geometry, observed that in addition to not being Gaussian, the returns process shows an interesting property: the time scale, that is, the distributions of returns. for various options of t, ranging from one day to one month, they have similar functional forms. As already mentioned, observed stock market prices are assumed to be the sum of many small terms, therefore a statistical model to describe them must be such that the sum of two or more independent random variables with a given identical distribution (with a parameter describing the tail drop) produce the same type of distribution. Motivated by these findings and empirical reasoning, Mandelbrot proposed that yields can be modeled as a kind of stable process introduced by Lévy in 1925, (Gauhtier-Villars et Cie, 1925). Stable tax distributions are attractive because they are supported by the generalized central limit theorem, which states that the laws are the only possible limit distributions for correctly normalized and centered sums of identically distributed independent random variables. Here an important problem is to know if the underlying distributions are really stable, since stability is only valid for $\alpha \in (0,2]$, some authors have discovered that the tails of some financial time series must be modeled with $\alpha > 2$.

In conclusion, the results on the distribution of returns are difficult to obtain and require a large amount of data to study extraordinary events that give rise to fat tails. Another problem is that, according to Cont (Cont R., 2001), for a parametric distributive model to reproduce the properties of the empirical distribution it must have at least four parameters: a location parameter, a scale parameter, a parameter that describe tailing decay and a skewness parameter within studies. It is known that there are alternative distributions that meet this condition, such as the Student's t or the inverse Gaussian. Therefore, in order to understand the universal laws behind the dynamics of financial markets, it is important to continue accumulating empirical data on different stock indices.

The objective of this paper is to provide a rigorous statistical analysis on the daily fluctuations of the logarithm of the returns of the main stock market indices of the main
emerging countries (Brazil, India, Hong Kong, China, Russia and Mexico). After considering the Gaussian, inverse Gaussian, and Lévy distributions, we find that the latter provides the best fit even though it is within the range of the family of stable distributions.

This brief is organized as follows. In the next section we will review: in section two the importance of the study on emerging markets, in section three the fundamental properties of Lévy alpha stable distributions. In section four, the data for the different indices are introduced and analyzed. In section five, the similarities and disagreements between the results obtained in this work and results from other authors will be discussed. Finally, in section five the conclusions will be presented.

2 EMERGING MARKETS

Emerging markets are those economies that are in a transition phase between developing countries and developed countries.

Investing in Emerging markets is more subtle and nuanced than simply investing in companies that enjoy certain profit prospects. Emerging markets are less homogeneous than developed markets, and are often at different stages of development.

“Global growth is slowing sharply, and further slowdown is likely as more countries enter recession. I worry that these trends will persist with lasting consequences that are devastating for people in emerging markets and developing economies,” said David Malpass, President of the World Bank Group. “To achieve low inflation rates, monetary stability, and faster growth, policymakers could shift the focus from reducing consumption to increasing production. Policies should aim to generate additional investment and improve productivity and capital allocation, which are critical for growth and poverty reduction.*World Bank/press release/January 2022.

Hence the importance of knowing the capital fluctuations that are projected for emerging countries such as Mexico, Brazil, Russia, Hong Kong and India for the coming years.

3 STABLE DISTRIBUTION

Stable Lévy distributions were first used by Mandelbrot (Mandelbrot, 1963), who postulated that cotton price fluctuations are a stochastic process with increments following a stable Lévy distribution. Stable distributions are characterized by having infinite variance and by meeting the stability condition, that is, the sum of identically
distributed random variables with a stable distribution remains a random variable with a stable distribution and the same functional form. This class of distributions are derived from a generalized version of the central limit theorem. An alpha stable Lévy distribution is characterized by a scale parameter, a skewness parameter, an exponent, and a location parameter. Since the analytic form of the stable distribution is known only in a few cases, it is generally specified by means of its characteristic function Levy and Khintchie (Mantegna and Stanley, 2000) found the general form of the characteristic function for stable distributions that has the shape:

\[
\ln \varphi(q) = \begin{cases} 
    i\mu q - \gamma |q|^\alpha \left[ 1 - i\beta \frac{q}{|q|} \tan \left( \frac{\pi \alpha}{2} \right) \right] & \text{para } \alpha \neq 1 \\
    i\mu q - \gamma |q|^\alpha \left[ 1 + i\beta \frac{q}{|q|} \frac{2}{\pi} \ln \left( \frac{\pi \alpha}{2} \right) \right] & \text{para } \alpha = 1
\end{cases}
\]

Later, other forms of the characteristic function for stable distributions were defined. These parameterizations were defined by G. Samorodnitsky and Taqqu (1994), and have the form:

\[
\Phi(t) = \begin{cases} 
    e^{-\gamma|t|^{\alpha} \left[ 1 + i \beta \frac{2}{\pi} \sin \left( \alpha \frac{\pi}{2} \right) \gamma(t) \ln|t| \right] + i\mu t} & \text{si } \alpha = 1 \\
    e^{-\gamma|t|^{\alpha} \left[ 1 - i \beta \tan \left( \frac{\alpha \pi}{2} \right) \sin(t) + i\mu t \right]} & \text{in another case}
\end{cases}
\]

where \( \sin g(t) \) denotes the sign of \( t \).

For stable distributions there is no general way to describe the density function of the distribution function, which are defined from the Fourier transform of the characteristic function.

\[
f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(t) e^{-itx} dt
\]

Where \( \varphi(t) \) is the characteristic function, which in the case of stable distributions has different parameterizations.
Lévy distributions are characterized by the property of being stable under convolution, that is, the sum of two or more independent random variables identically distributed by Lévy, also has a Lévy distribution with the same stability index. The stability parameter is in the interval (0, 2]. Presenting a sharp peak due to the heavy tails that decay asymptotically as power laws with an exponent $-(\alpha + 1)$. For the normal distribution $\alpha = 2$ and parameter of asymmetry $\beta = 0$.

As already mentioned, a stable distribution is characterized by four parameters, an exponent or parameter known as the stability index, a scale parameter, a skewness or bias parameter, and a position or location parameter. These parameters can be interpreted as follows:

- **Characteristic exponent:** This parameter takes values in the interval $(0, 2]$. In particular when $\alpha = 2$ there is a normal distribution. We can think that this parameter indicates a departure from normality, that is, the distribution analyzed will move away from the normality as it takes values further from 2. It also gives an idea of the decay of the distribution when $x$ takes very large values.

- **Scale parameter:** It is incorrect to interpret the scale parameter as the standard deviation of the process since it exists only for the case of $\alpha = 2$. However, the larger the value of the wider the density function.

- **Skewness parameter:** The parameter indicates how skewed the density function is. That is, the parameter takes values that belong in the interval $[-1, 1]$. That is, when $\beta = 1$ the density function is totally skewed to the right, while if $\beta = -1$ the density is skewed to the left and only when $\beta = 0$ is there a symmetric case.

- **Location parameter:** For the case of the location parameter, $\mu = E(X)$ in the case that the first moments exist.

The asymptotic behavior of Lévy distributions is described by the expression.

$$f(x; \alpha) \approx |x|^{-1-\alpha}$$

Therefore, the variance of stable Lévy distributions is infinite for all $\alpha < 2$. In these calculations, the "STABLE" library developed in MATLAB by J.P. Nolan (Nolan, et al, 2009).
4 STATISTICAL DATA ANALYSIS

Within this section, we will analyze the financial indices of the main emerging countries, applying the techniques described above. The analysis covers a period of 22 years (period mentioned above). The different indices have been labeled as $Y(t)$ and the data written as the successive differences of the natural logarithm of the yields.

$$S(t) = \ln Y(t + \Delta t) - \ln Y(t)$$

Closing daily values were used, so $t = 1$. Graph 2 shows the value of the index as a function of the trading day and the corresponding histogram for $S(t)$ is plotted.

Table 1. Shows the value of the calculation of important statistics for the indices analyzed. The high value of kurtosis shows that the density functions of the time series have more peaks than in the Gaussian distributions.

<table>
<thead>
<tr>
<th>Country analyzed</th>
<th>Index</th>
<th>Average</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>IPC</td>
<td>$3.0958 \times 10^4$</td>
<td>$2.4141 \times 10^8$</td>
<td>-0.4363</td>
<td>-1.2762</td>
</tr>
<tr>
<td>Brazil</td>
<td>Bovespa</td>
<td>$4.7892 \times 10^4$</td>
<td>$2.5755 \times 10^4$</td>
<td>3.5772</td>
<td>10.9830</td>
</tr>
<tr>
<td>India</td>
<td>Nifty 50 (NSEI)</td>
<td>$6.2552 \times 10^4$</td>
<td>$1.9413 \times 10^4$</td>
<td>0.8283</td>
<td>0.0261</td>
</tr>
<tr>
<td>Russia</td>
<td>IMOEX Russia</td>
<td>$1.5168 \times 10^5$</td>
<td>$8.3838 \times 10^3$</td>
<td>0.5364</td>
<td>0.0685</td>
</tr>
<tr>
<td>China</td>
<td>Shanghai Composite (SSEC)</td>
<td>$2.5652 \times 10^5$</td>
<td>$7.7499 \times 10^3$</td>
<td>0.5495</td>
<td>0.5521</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>HangSeng (HSI)</td>
<td>$2.0465 \times 10^4$</td>
<td>$3.3228 \times 10^4$</td>
<td>-0.3236</td>
<td>-0.8400</td>
</tr>
</tbody>
</table>
4.1 GAUSSIAN FIT

Within probability theory, an important area is focused on parametric estimation (Inferential Statistics), which indicates the purpose of carrying out or using estimation theory to arrive at an estimator or preferably a result that is reliable in making certain decisions as the case may be. To obtain the estimator, data such as the daily closing values of financial shares are taken into account.

To estimate the parameters of a stable Lévy distribution, there are methods such as: 1) the Maximum Likelihood (ML) method, (2) the quantile or moment tabulation method, and (3) the regression method of the characteristic function. For this research work, the Maximum Likelihood method will be applied, which provides greater efficiency, consisting of parametric estimation. For the analysis of stable distributions, the previously mentioned library was used to develop this task.

In general, this method allows calculating the value of the parameter that increases the function to its maximum likelihood. The ML method is asymptotically consistent, that is, as the sample size increases, the parameter value estimate converges faster to a correct value. For stable distributions, both the pdf and the probability distribution function are not known explicitly. A proposal for the solution to this problem is a numerical approximation of the method, which was initially applied to stable distributions by DuMouchel (DuMouchel, 1973). To illustrate the method, consider a sample composed of the time series of finite size denoted by If i = 1N in this way, assuming that our sample data comes from identically distributed random variables, then the maximum likelihood function can be denoted as:

\[ L(\alpha, \beta, \gamma, \delta_0) = \sum_{i=1}^{N} \log S_0(S_i | \alpha, \beta, \gamma, \delta_0) \]

Where \( \delta_0, \beta, \alpha, \gamma \), is the probability function for a stable alpha distribution, with the parameterization S0. The values for the parameters of the above equation are chosen as the best estimate of the stable distribution. To carry out the estimation of the parameters we use the STABLEFIT function, which is integrated into the STABLE library developed by Nolan (Nolan, 2009). Table 2 shows the results for the calculated statistics.
Within the same study area, there are various goodness-of-fit tests, for this research the Kolmogorov - Smirnov (K - S) criterion was applied, which will be very useful. In general, goodness-of-fit tests measure the comparability or the relationship that can exist between data from a random sample with a probability distribution function. In other words, doing a goodness-of-fit test usually involves examining the sample with some unknown distribution to test the contrast hypothesis regarding the proposed distribution, if the test is correct, it is mentioned that there is a distribution that models so sample appropriate. It can be assumed that the goodness-of-fit method consists of defining a test statistic, that is, a function is defined that measures the distance between the hypothesized distribution and the sample data, then the probability that the data is have a value greater than the test statistic and the observed value. Assuming that the hypothesis is true, this probability is considered as the level of confidence to consider.

There are various methods or tests that are important when applying the goodness-of-fit criterion, among the most important methods are the Pearson test denoted by $x^2$, since it is considered a non-parametric test that measures the discrepancy between a distribution observed and a theoretical one indicating to what extent the differences

### Table 2. Estimation of the parameters for the alpha stable, normal and inverse Gaussian distributions.

<table>
<thead>
<tr>
<th></th>
<th>IPC</th>
<th>Ibovespa</th>
<th>Nifty 50</th>
<th>IMOEX</th>
<th>Shanghai Composite</th>
<th>Hang-Seng</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gaussian fit</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$-stable fit</td>
<td>1.4092</td>
<td>-0.6081</td>
<td>0.0061</td>
<td>0.0031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NIG fit</td>
<td>65.0579</td>
<td>1.609</td>
<td>0.0093</td>
<td>2.1388 $\times 10^{-04}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gaussiana fit</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$-stable fit</td>
<td>1.4308</td>
<td>0.1971</td>
<td>0.0513</td>
<td>-0.0038</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NIG fit</td>
<td>43.1655</td>
<td>0.3778</td>
<td>0.0103</td>
<td>-4.0874 $\times 10^{-04}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Nifty 50</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$-stable fit</td>
<td>1.6258</td>
<td>0.1872</td>
<td>0.0074</td>
<td>-0.0011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NIG fit</td>
<td>440812</td>
<td>0.7898</td>
<td>0.109</td>
<td>1.7857 $\times 10^{-04}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>IMOEX</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$-stable fit</td>
<td>1.5585</td>
<td>0.1219</td>
<td>0.0093</td>
<td>-0.0012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NIG fit</td>
<td>87.0691</td>
<td>0.0964</td>
<td>0.8258</td>
<td>-6.0746 $\times 10^{-04}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Shanghai Composite</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$-stable fit</td>
<td>1.5566</td>
<td>0.096</td>
<td>0.0076</td>
<td>-0.0007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NIG fit</td>
<td>65.6063</td>
<td>-6.0852</td>
<td>0.7155</td>
<td>0.9823</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Hang-Seng</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$-stable fit</td>
<td>1.6506</td>
<td>0.1528</td>
<td>0.0077</td>
<td>-0.0006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NIG fit</td>
<td>46.0271</td>
<td>-1.0627</td>
<td>0.0054</td>
<td>3.2891 $\times 10^{-04}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
between them are due to chance in the contrast of hypotheses. Something important is to note that in Pearson's test, the greater the value of $x_2$, the less likely it will be to assume equality between both distributions. As well as the Pearson test, we have the Anderson-Darling test, which is a non-parametric test that is generally used to compare the fit of an observed cumulative distribution function with an expected cumulative distribution function. The latter is used to test whether a data sample comes from a population with a known distribution. The Anderson-Darling test is often said to be a modification of the K-S test, since it gives more weight to the distribution tails.

The K-S test is distribution-independent in the sense that critical values do not depend on a specific distribution to be tested. In addition to being a non-parametric test that is used to assess whether or not a sample comes from a population, on the other hand, the K-S test is based on calculating the maximum distance between the cumulative function of the theoretical distribution and the function of empirical distribution fitted from the sample.

The K-S test (Gnedenko, 2018) is used to decide if a sample comes from a population with a specific distribution. This test is based on the Empirical Distribution Function (ECDF). Given a set of N ordered data, $X_1, X_2, X_3, \ldots, X_n$, the ECDF is defined as:

$$E_n = \frac{i}{n}$$

where $i$ is the number of points smaller than $x_i$ and the $x_i$ whose values are ordered from smallest to largest. This is a step function that increases by $1/N$ for each sorted data value. The experimental statistic for the K-S test can be obtained by ordering the data in ascending order $X_1, X_2, X_3, \ldots, X_n$, and deriving the maximum difference between the order statistic and the CDF $F(x_i)$:

$$D = \max \left[ \max \left\{ \frac{i-1}{n}, \left| \frac{i}{n} - F(x_i) \right| \right\} \right]$$

La prueba de Kolmogorov-Smirnov es definida por:
• The null hypothesis $H_0$: The analyzed data follows a specific distribution.

• Alternative Hypothesis $H_1$: The analyzed data does not follow the specified distribution.

The null hypothesis $H_0$ (the analyzed data follows a specific distribution) is rejected if the test statistic, $D$, is greater than the critical value (for a level of confidentiality obtained from statistical tables (Gnedenko, 2018). There are variations of these tables in the literature that use somewhat different scales for the test statistic and critical regions. These alternative formulations should be equivalent, but it is necessary to ensure that the test statistic is calculated in a way that is consistent with the way it is calculated. Critical values were tabulated. An attractive feature of this test is that the distribution of the K-S test statistic itself does not depend on the underlying cumulative distribution function being tested.

Since in this study the parameters of the distributions were fitted from the observed data, the theoretical critical values provided by the K-S criterion cannot be used (Gnedenko, 2018). In this case, approximate p-values can be obtained by Monte Carlo simulations, by the method described in Kowaka et al. (1994).

First, the parameter vector is estimated for a given sample of size $n$, being the result, and the test statistics are computed assuming the sample is distributed according to $F_X$, returning the value of $D$. Next, a sample of size $n$ of $F_X$ is generated, and the parameter vector 1 is estimated. The test statistics are again calculated assuming that the sample is distributed according to $F_X$; he. The simulation is repeated 1000 times and the estimate of the p-value is calculated as the relative number of occasions in which the test statistics are greater than $D$.

In this study, the Kolmogorov-Smirnov test (with a confidentiality level of 5%) was applied to test whether price fluctuations follow an alpha-stable, Gaussian and Inverse Gaussian distribution. Next, tables 3, 4 and 5 show the results of the critical values that were calculated by means of the K-S criterion, considering the stable alpha distribution of Lévy, the inverse Gaussian distribution and the normal distribution, without forgetting that these results they are obtained through the distributions that were fitted from the data, similarly the limit values were estimated from Monte Carlo (M-C) simulations.
**Table 3. Results of the Kolmogorov-Smirnov test for stable alpha distribution (for 5% confidence, α=0.05).**

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Stock index</th>
<th>Theoretical critical values for the K-S distribution</th>
<th>Critical values estimated by Monte Carlo simulation</th>
<th>Test Statistic for Stable Distribution</th>
<th>P-Value for stable distribution</th>
<th>Decline - $H_0$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>5768</td>
<td>IPC</td>
<td>0.0271</td>
<td>0.0208</td>
<td>0.0164</td>
<td>0.1331</td>
<td>No</td>
</tr>
<tr>
<td>5768</td>
<td>Ibovespa</td>
<td>0.027</td>
<td>0.0207</td>
<td>0.0182</td>
<td>0.1974</td>
<td>No</td>
</tr>
<tr>
<td>5659</td>
<td>Nifty 50 (NSEI)</td>
<td>0.0308</td>
<td>0.0231</td>
<td>0.019</td>
<td>0.201</td>
<td>No</td>
</tr>
<tr>
<td>5680</td>
<td>IMOEX RUSSIA</td>
<td>0.0269</td>
<td>0.0206</td>
<td>0.0145</td>
<td>0.1201</td>
<td>No</td>
</tr>
<tr>
<td>5516</td>
<td>ShanghaiComposite (SSEC)</td>
<td>0.0275</td>
<td>0.0185</td>
<td>0.0125</td>
<td>0.1133</td>
<td>No</td>
</tr>
<tr>
<td>5408</td>
<td>HangSeng (HSI)</td>
<td>0.027</td>
<td>0.019</td>
<td>0.0159</td>
<td>0.4379</td>
<td>No</td>
</tr>
</tbody>
</table>

**Table 4. Results of the Kolmogorov-Smirnov test for the Gaussian distribution (for 5% confidence, α=0.05).**

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Stock index</th>
<th>Theoretical critical values for the K-S distribution</th>
<th>Critical values estimated by Monte Carlo simulation</th>
<th>Test Statistic for Stable Distribution</th>
<th>P-Value for stable distribution</th>
<th>Decline - $H_0$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>5768</td>
<td>IPC</td>
<td>0.0271</td>
<td>0.0184</td>
<td>0.0654</td>
<td>0.0008</td>
<td>SI</td>
</tr>
<tr>
<td>5768</td>
<td>IBOVESPA</td>
<td>0.027</td>
<td>0.0182</td>
<td>0.0476</td>
<td>0.0036</td>
<td>SI</td>
</tr>
<tr>
<td>5659</td>
<td>Nifty 50 (NSEI)</td>
<td>0.0308</td>
<td>0.0201</td>
<td>0.0509</td>
<td>0.0098</td>
<td>SI</td>
</tr>
<tr>
<td>5680</td>
<td>IMOEX RUSSIA</td>
<td>0.0269</td>
<td>0.0206</td>
<td>0.0465</td>
<td>0.003</td>
<td>SI</td>
</tr>
<tr>
<td>5516</td>
<td>Shanghai Composite (SSEC)</td>
<td>0.0275</td>
<td>0.0185</td>
<td>0.0355</td>
<td>0.0051</td>
<td>SI</td>
</tr>
<tr>
<td>5408</td>
<td>HangSeng (HSI)</td>
<td>0.027</td>
<td>0.019</td>
<td>0.0479</td>
<td>0.0072</td>
<td>SI</td>
</tr>
</tbody>
</table>

**Table 5. Results of the Kolmogorov-Smirnov test for the inverse Guassina distribution (for 5% confidence, α=0.05).**

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Stock index</th>
<th>Theoretical critical values for the K-S distribution</th>
<th>Critical values estimated by Monte Carlo simulation</th>
<th>Test Statistic for Stable Distribution</th>
<th>P-Value for stable distribution</th>
<th>Decline - $H_0$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>5768</td>
<td>IPC</td>
<td>0.0272</td>
<td>0.0207</td>
<td>0.0271</td>
<td>0.01</td>
<td>SI</td>
</tr>
<tr>
<td>5768</td>
<td>Ibovespa</td>
<td>0.027</td>
<td>0.0208</td>
<td>0.027</td>
<td>0.0106</td>
<td>SI</td>
</tr>
<tr>
<td>5659</td>
<td>Nifty 50 (NSEI)</td>
<td>0.0214</td>
<td>0.0207</td>
<td>0.0214</td>
<td>0.001</td>
<td>SI</td>
</tr>
<tr>
<td>5680</td>
<td>IMOEX RUSSIA</td>
<td>0.0269</td>
<td>0.0212</td>
<td>0.0246</td>
<td>0.0101</td>
<td>SI</td>
</tr>
<tr>
<td>5516</td>
<td>Shanghai Composite (SSEC)</td>
<td>0.027</td>
<td>0.0285</td>
<td>0.0269</td>
<td>0.0103</td>
<td>SI</td>
</tr>
<tr>
<td>5408</td>
<td>HangSeng (HSI)</td>
<td>0.0271</td>
<td>0.0205</td>
<td>0.0271</td>
<td>0.0109</td>
<td>SI</td>
</tr>
</tbody>
</table>
As can be seen in Tables 3-5, the critical values estimated from Monte Carlo simulations are always lower than the theoretical critical values, so we are in the presence of a much more rigorous goodness-of-fit test than for the case of an unknown distribution.

5 CONCLUSIONS

As can be observed throughout this research work, the Gaussian distribution is not a good model to describe the fluctuations of the daily data of the financial markets. In this case, for all the indexes studied, the null hypothesis (the fluctuations of the analyzed data follow a Gaussian distribution) is rejected by applying the Kolmogorov-Smirnov test. Therefore, it can be said that the Brownian movement is not the appropriate tool to model financial indexes for emerging countries. The stable Lévy distribution has been shown to be a more reliable model for adjusting for daily data fluctuations. This can be assured because statistical tests support this idea.

A statistical analysis was carried out for the daily data considering a period of twenty-two years (2000-2022), and the behavior of the main index of some emerging countries such as: China with the Shanghai Composite Index (SSEC), Brazil with the Bovespa index (IBOVESPA), India with the Nifty50 index (NSE), Russia with the MOEX index (IMOEX), Hongkong with the Hang Seng index (HSI), in addition to the (IPC) for Mexico. Through this analysis, it can be observed that for a daily horizon and considering a 5% confidence, the fluctuations of the stock market index of the countries analyzed (Mexico, Brazil, Russia, India, China and Hong Kong) are adequately modeled from of a stable Lévy distribution. Considering these results, as a conclusion we can say that, in the case of daily data, the Lévy processes provide an adequate description of market fluctuations for the BRICs and Mexico. The normality hypothesis is rejected for all the indexes analyzed.
REFERENCES


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